BEATTY SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2010

SUBJECT	: Mathematics	**LEVEL**	: Sec 4E / 4N2 / 5N
PAPER	: 4016/2	**DURATION**	: 2 hours 30 minutes
SETTER	: Mdm Rashima Sidik	**DATE**	: 20 September 2010

CLASS :	**NAME :**	**REG NO :**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces on the top of this page.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **100**.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 **(a)** Simplify $\dfrac{25-a^2}{3a^2+13a-10}$. [3]

(b) **(i)** Make v the subject of the formula $\sqrt{\dfrac{w-v}{v}}=\dfrac{1}{u}$. [3]

(ii) Find u when w = –15 and v = –3. [2]

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2 **(a)** Nora receives a salary of $49 500 per annum. The first $20 000 is not taxable. A tax rate of 15% is imposed on the remaining income. Calculate her average **monthly** income after paying her tax. [2]

(b) For her HDB flat, Nora has to make monthly installments of a 30-year home loan of $365 000 at 2.6 % per annum simple interest. How much does she have to pay for her monthly installment? [3]

(c) Nora has RM120 left over from her holiday trip. How much Singapore dollars can she exchange for, given the current exchange rate as follows: [2]

		Singapore Dollars	
Currency	**Unit**	**Selling**	**Buying**
Malaysian Ringgit	1	0.427	0.431

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3 Before the price increase, a chef can buy x kg of flour for $78. After the price increase, he will get 2 kg less of flour for the same amount of money.

(a) Write down an expression, in terms of x, for the price of flour per kilogram,

(i) before the increase in price, [1]

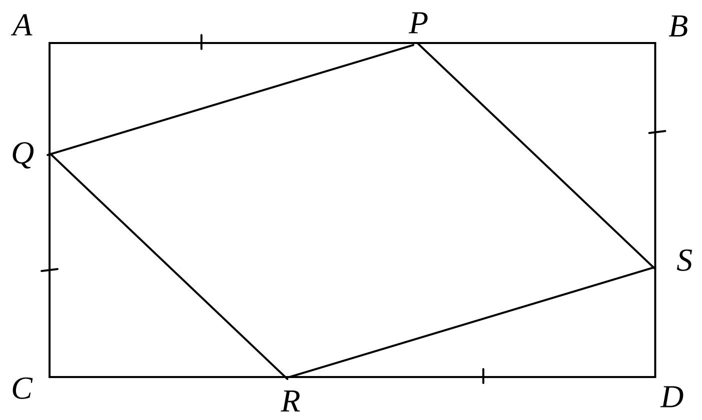
(ii) after the increase in price. [1]

(b) If the increase in price is 9 cents per kilogram of flour, form an equation in x and show that it reduces to $3x^2-6x-5200=0$. [3]

(c) Solve the equation $3x^2-6x-5200=0$, giving your answers correct to 2 decimal places. [2]

(d) Hence, find the price per kilogram of flour after the price increase. [2]

4



In the diagram, *ABCD* is a rectangle. Points *P*, *Q*, *R* and *S* lie on *AB*, *AC*, *CD* and *BD* so that *CQ* = *AP* = *BS* = *RD*.

(a) Stating your reasons, prove that

(i) $AQ = SD$, [2]

(ii) ΔQAP is congruent to ΔSDR. [2]

(b) State another pair of congruent triangle and hence prove that *PQRS* is a parallelogram. [2]

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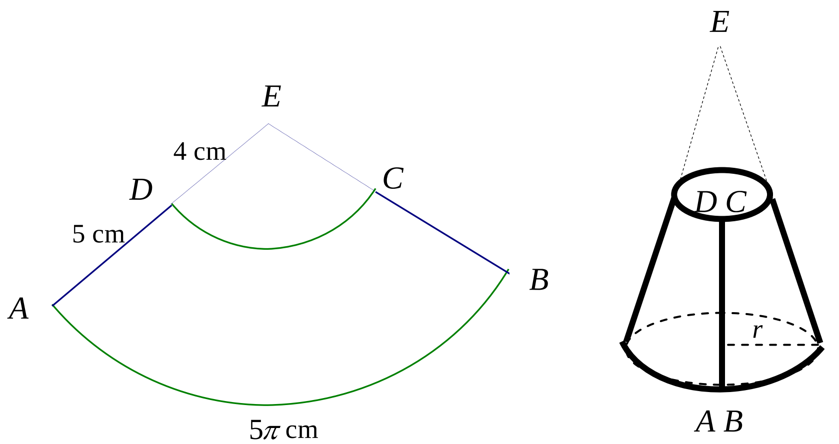
5 In the diagram, a smaller sector *EDC* is cut out from a larger sector *EAB* to form the figure *ABCD*. *AD* = 5 cm, *DE* = 4 cm and arc *AB* = 5π cm.



(a) Leaving your answers in π, find

(i) $\angle AEB$ in radians, [1]

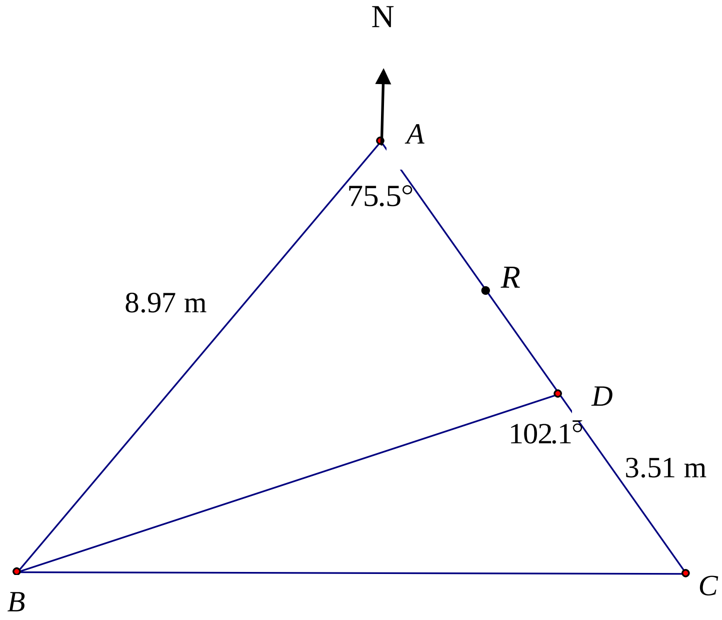
(ii) the perimeter of the figure *ABCD*. [3]

(b) *ABCD* is wrapped to form a frustum where the side *AD* meets side *BC*.

(i) Show that the radius, *r*, of the lower base of the frustum, is 2.5 cm. [2]

(ii) Find the volume of the frustum. [4]

6 The diagram shows points A, B, C, D and R on level ground of a home garden, with C due east of B. $\angle BDC = 102.1°$, $\angle BAD = 75.5°$, $AB = 8.97$ m and $CD = 3.51$ m.



(a) Calculate

(i) the length of BD, [2]

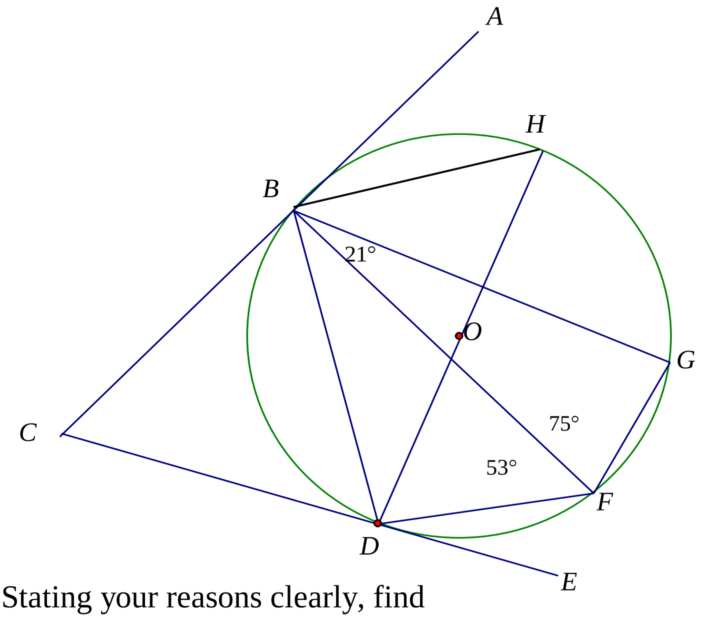
(ii) the length of BC, [2]

(iii) the bearing of D from B. [3]

(b) A vertical pole of 15.2 m stands at A. Calculate the angle of elevation of the top of the pole from B. [2]

(c) Alex walked from A towards D and stopped at a point R, where the area of the triangle ARB is 17.5 m^2. Find the length of AR. [2]

7 In the diagram, DOH is the diameter of the circle with centre O. ABC and CDE are tangents to the circle. $\angle FBG = 21°$, $\angle BFD = 53°$ and $\angle BFG = 75°$.



(a) Stating your reasons clearly, find

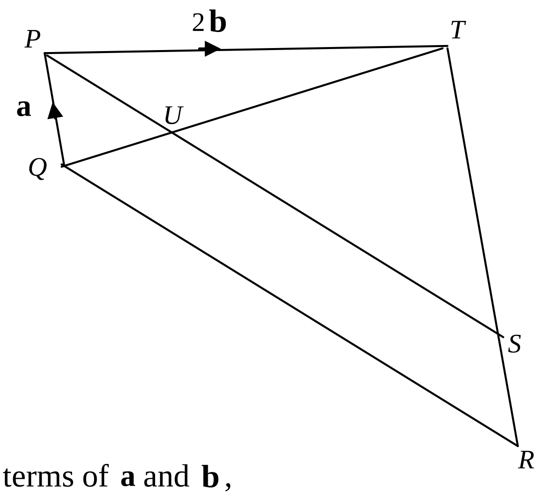
(i) $\angle DBF$, [2]

(ii) $\angle HDB$, [2]

(iii) $\angle BCD$, [2]

(b) Given that CD = 10 cm, find the radius of the circle. [3]

8 In the diagram, *PQRS* is a parallelogram and *RST* lie on a straight line.
$\overrightarrow{QP} = \mathbf{a}$, $\overrightarrow{PT} = 2\mathbf{b}$ and $\overrightarrow{RT} = 4\overrightarrow{RS}$.



(a) Express, in terms of $\mathbf{a}$ and $\mathbf{b}$,

(i) the vector $\overrightarrow{QT}$, [1]

(ii) the vector $\overrightarrow{PS}$. [1]

(b) It is given that $\overrightarrow{QU} = m\overrightarrow{QT}$. Express and simplify the vector $\overrightarrow{QU}$, in terms of $\mathbf{a}$, $\mathbf{b}$ and m. [1]

(c) It is also given that $\overrightarrow{PU} = n\overrightarrow{PS}$. Express and simplify the vector $\overrightarrow{PU}$, in terms of $\mathbf{a}$, $\mathbf{b}$ and n. [1]

(d) Find the values of m and n. [3]

(e) State the ratio $\dfrac{area\ of\ \Delta PQU}{area\ of\ \Delta PQT}$. [1]

9 Table 1 shows the number of packets of rice of four brands; *A*, *B*, *C* and *D*, sold at 3 grocery shops.

Table 1

	Brand *A*	Brand *B*	Brand *C*	Brand *D*
Shop 1	5	3	1	1
Shop 2	11	6	8	5
Shop 3	8	5	4	3

A packet of rice costs \$15 for Brand *A*, \$10 for Brand *B*, \$8 for Brand *C* and \$5 for Brand *D*.

(a) Given the matrix $\mathbf{P} = \begin{pmatrix} 15 \\ 10 \\ 8 \\ 5 \end{pmatrix}$,

(i) Write down a 3×4 matrix $\mathbf{Q}$ that represents the data in Table 1. [1]

(ii) Evaluate $\mathbf{QP}$. [1]

(iii) Explain what the elements in $\mathbf{QP}$ represent. [1]

Brand *A* comes in packets of 8-kg, Brand *B* in packets of 10-kg, Brand *C* in packets of 7-kg and Brand *D* in packets of 9-kg.

(b) **(i)** Write down a 4×1 matrix $\mathbf{S}$ such that matrix $\mathbf{QS}$ represents the total number of kilograms of rice sold by each shop. [1]

(ii) Evaluate $\mathbf{QS}$. [1]

(iii) Hence deduce the average cost of rice per kilogram for Shop 2. [2]

10 Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation $2y = x - \frac{5}{x}$.

The table below shows some corresponding values of x and y, correct to 2 decimal places.

x	0.5	1	1.5	2	3	3.5	4
y	– 4.75	– 2	– 0.92	a	0.67	1.04	1.38

(a) Show that $a = -0.25$. [1]

(b) Using a scale of 4 cm to represent 1 unit on the x-axis and 2 cm to represent 1 unit on the y-axis, draw the graph of $2y = x - \frac{5}{x}$ for $0 \le x \le 4$. [3]

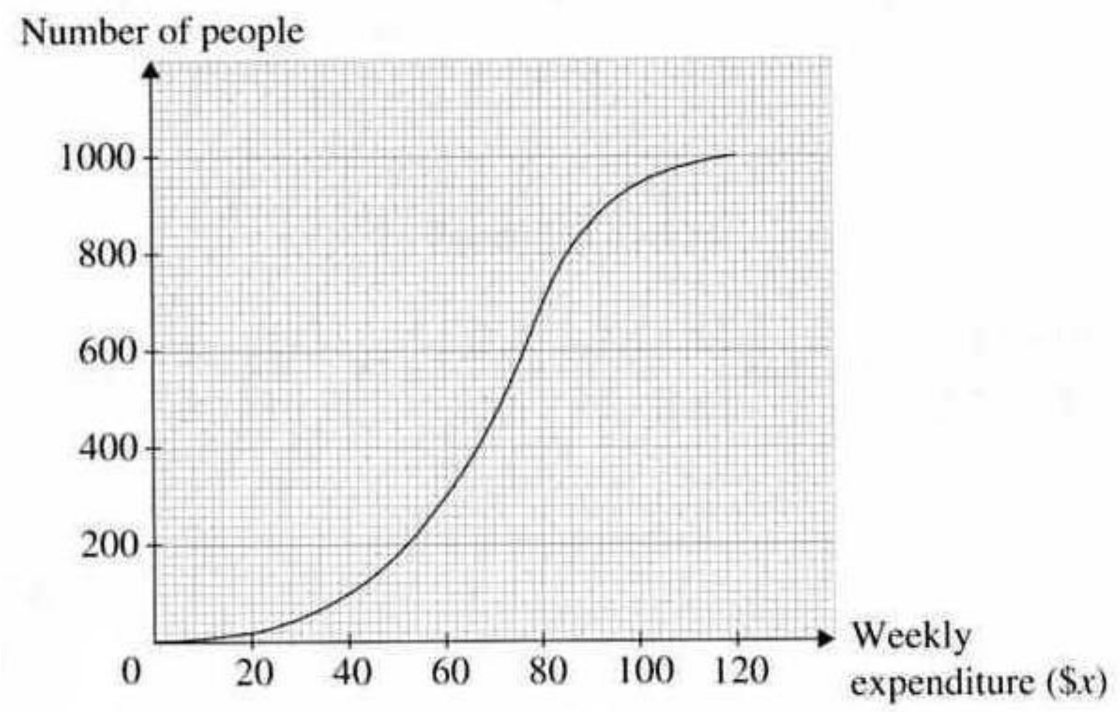
(c) Use your graph to find the solution to the equation $x - \frac{5}{x} = 0$. [1]

(d) By drawing a suitable straight line on the graph, find the solution to the equation $x - \frac{5}{x} = -\frac{2}{3}x + 2$. [2]

(e) By drawing a tangent, find the gradient of the curve at $x = 1$. [2]

(f) State the coordinates of the point Q on the curve where the gradient of the tangent is 1. [1]

11 **(a)** The cumulative frequency curve shows the weekly expenditure of 1000 people.



(i) Copy and complete the frequency table below. [2]

Weekly expenditure ($x)	$0 < x \leq 20$	$20 < x \leq 40$	$40 < x \leq 60$	$60 < x \leq 80$	$80 < x \leq 100$	$100 < x \leq 120$
Number of people						

(ii) Use the graph to find the

(a) interquartile range, [2]

(b) the number of people whose weekly expenditure is more than $82.00. [2]

(iii) Using a scale of 1 cm for $10 for the weekly expenditure and 2 cm for 50 people for the number of people, draw a histogram to represent the distribution. [2]

(b) At a carnival game, John throws two dice and the score is the sum of the numbers shown on the dice. If the score is 10 or more, he wins the game. Otherwise, he loses.

(i) Copy and complete the possibility diagram below. [2]

SUM		Die 1					
		1	2	3	4	5	6
Die 2	1						
	2						
	3						
	4						
	5						
	6						

(ii) Show that the probability that John wins the game is $\frac{1}{6}$. [1]

(iii) If John plays the game twice,

(a) Draw a tree diagram to show the possible outcomes of the two games. [2]

(b) Find the probability that John loses at least one game. [2]

Answer key

Q1a) $\dfrac{5-a}{3a-2}$

bi) $v=\dfrac{wu^2}{1+u^2}$

ii) $u=\dfrac{1}{2}$

Q2a) \$3756.25

b) \$1804.72

c) \$51.24

Q3ai) $\$\dfrac{78}{x}$

ii) $\$\dfrac{78}{x-2}$

bi)
$$\frac{78}{x-2}-\frac{78}{x}=\frac{9}{100}$$
$$\frac{78x-78(x-2)}{x(x-2)}=\frac{9}{100}$$
$$15600=9x^2-18x$$
$$3x^2-6x-5200=0\text{(shown)}$$

c) \$ 42.65

d) \$1.92

Q4ai) $\left.\begin{array}{l}\text{AC=BD (given)}\\ \text{QC=BS (given)}\end{array}\right\}$ M1

$\therefore AC-QC=BD-BS$

$AQ=SD$ (proven)

ii) AQ = SD (proven in (ai))

$\angle PAQ=\angle SDR=90°$

AP=RD (given)

$\therefore \Delta QAP\equiv\Delta SDR$ **(SAS test)** (proven)

b) Another pair of congruent Δs:

ΔRCQ *and* ΔPBS

$\Rightarrow RQ=PS$

$\&\ PQ=RS$

$\therefore PQRS$ is a parallelogram. (proven)

5ai) $\frac{5\pi}{9} rad$

ii) $10+\frac{65}{9}\pi$

bi) $2\pi r=5\pi$

r = $\frac{5\pi}{2\pi}$

=2.5 cm (shown)

ii) 51.6 cm^3

6ai) $BD=8.88m$

ii) $BC\approx 10.2m$

iii) 070.4 °

b) 59.5°

c) 4.03m

7ai) 31°

ii) 37°

iii) 74°

b) 7.54 cm

8ai) $\underset{\sim}{a}+2\underset{\sim}{b}$

ii) $2\underset{\sim}{b}-3\underset{\sim}{a}$

b) $m(\underset{\sim}{a}+2\underset{\sim}{b})$

c) $n(2\underset{\sim}{b}-3\underset{\sim}{a})$

d) m = $\frac{1}{4}$, n = $\frac{1}{4}$

e) $\frac{1}{4}$

9*ai*) $\begin{pmatrix} 5 & 3 & 1 & 1 \\ 11 & 6 & 8 & 5 \\ 8 & 5 & 4 & 3 \end{pmatrix}$

ii) $\begin{pmatrix} 118 \\ 314 \\ 217 \end{pmatrix}$

iii) They represent the **total amount of money** (in dollars) **each shop received** from selling the rice packets respectively.

bi) $\begin{pmatrix} 8 \\ 10 \\ 7 \\ 9 \end{pmatrix}$

ii) $\begin{pmatrix} 86 \\ 249 \\ 169 \end{pmatrix}$

iii) $1.26

10a) $a = \dfrac{2 - \frac{5}{2}}{2}$

$= -0.25$ (*shown*)

c) $x = 2.24 \pm 0.05$

d) $x = 2.43 \pm 0.05$

e) Gradient at (1,-2) = 3 (±0.5)

f) Q (2.25, 0) (±0.1)

11ai)

Weekly Expenditure($)	0<x≤20	20<x≤40	40<x≤60	60<x≤80	80<x≤100	100<x≤120
No. of people	20	80	200	400 or 380	240 or 260	60

iia) $27 (±2)

iib) 260 (±20)

bi)

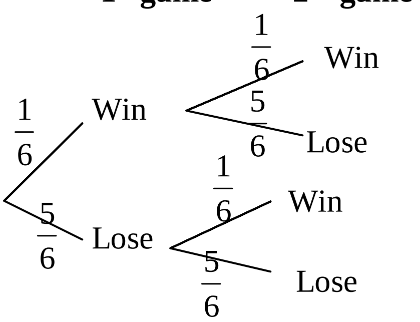
SUM		Die 1					
		1	2	3	4	5	6
Die 2	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

ii) P(player wins a game)

$= \frac{6}{36}$

$= \frac{1}{6} (\textit{shown})$

iiia) **1^{st} game** **2^{nd} game**



b) $\frac{35}{36}$